Numerical Analysis and Optimization

Course Syllabus

Basic Information

Program of study	Applied Computing in Engineering and Science (Master's Degree)
Semester	Second (Year 1)
Course credits	6 ECTS
Language	English level B1 / Intermediate (European Framework of Reference of Communicative Skills)
Prerequisites	B. Sc. degree in Mathematics, Physics or Computer Science

Course Instructor

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Required Knowledge

The students who take this course should have a stable knowledge of single and multi-variable calculus, linear algebra and be familiar with basic facts from the theory of differential equations. Elementary level of programming knowledge is required. The knowledge of a high-level programming language such as C++, Fortran would help, but is not required.

Preliminary Reading:

1. G. Mclelland. On the shoulders of giants: A course in single variable calculus. University of New South Wales Press, 2002, ISBN 0868407178.
2. T. M. Apostol. Calculus, Vol. 1: One-Variable Calculus with an Introduction to Linear Algebra. John Wiley & Sons, Inc., 1967, ISBN 0471000051.
3. T. M. Apostol. Calculus, Vol. 2: Multi-Variable Calculus and Linear Algebra, with Applications to Differential Equations and Probability. John Wiley & Sons, Inc., 1969, ISBN 9780471000075.
4. L. Hogben (editor). Handbook of Linear Algebra. Boca Raton, Florida: CRC Press/Taylor & Francis Group, 2014, ISBN 9781466507289.
5. B. W. Kernighan, R. Pike: The practice of programming, Reading, MA: Addison-Wesley, 1999, ISBN 020161586X.

Computing

Access to one of the computing environments such as C++, Fortran, Maple, Matlab, Mathematica, etc. is required in order to do the computer assignments.

Teaching

The course will be taught mainly in lecture format along with class examples and demonstrations. Individual Tasks (Computer Assignments) will be assigned. At the end of the terms the qualification (oral examination) will take place.

Course Description

The course emphasizes the underlying fundamental ideas behind numerical methods and covers important topics, such as the basics of the analysis of algorithms and computational complexity. The first part of the course introduces the necessary mathematical background, the digital representation of numbers, and different types of errors associated with numerical methods. The second part explains how to solve typical problems using numerical methods. Focusing on optimization methods, the final part presents basic theory and algorithms for linear and nonlinear optimization.

Course Aims

The aim of the course is to present a creation, analyzing, and implementation algorithms for

- obtaining numerical solutions to problems of calculus;
- selection of a best element (with regard to some criteria) from some set of available alternatives.

Such problems arise throughout the natural sciences, social sciences, engineering, medicine, and business.

Course Objectives

The objective of the course is teach to

- find acceptable approximate solutions when exact solutions are either impossible or so arduous and time-consuming as to be impractical;
- devise alternate methods of solution better suited to the capabilities of computers;
- formulate problems in their fields of research as optimization problems by defining the underlying independent variables, the proper cost function, and the governing constraint functions;
- understand how to assess and check the feasiblity and optimality of a particular solution to a general constrained optimization problem;
- use the optimality conditions to search for a local or global solution from a starting point;
- formulate the dual problem of some general optimization types and assess their duality gap using concepts of strong and weak duality;
- understand the computational details behind the numerical methods discussed in class, when they apply, and what their convergence rates are.

Special Features

The course deals with the algorithms for obtaining numerical solutions to problems involving continuous variables. Such problems arise throughout the natural sciences, social sciences, engineering, medicine, and business. Since the mid 20th century, the growth in power and availability of digital computers has led to an increasing use of realistic mathematical models in science and engineering, and numerical analysis of increasing sophistication is needed to solve these more detailed models of the world.

Moreover, the course presents a comprehensive and up-to-date description of the most effective methods in continuous optimization. It responds to the growing interest in optimization in engineering, science, and business by focusing on the methods that are best suited to practical problems.

Outline of Content

1. Number Systems and Errors.
2. Solving Equations.
3. Polynomial Interpolation and Numerical Integration.
4. Numerical Solution of Differential Equations.
5. Basic Concepts of Numerical Optimization: Formulating an Optimization Problem, Local and Global Optimality, Existence of an Optimal Solution, Level Sets, Gradients, Convex Sets.
6. Complexity Issues of Numerical Optimization: Algorithms and Complexity, Average Running Time, Randomized Algorithms, Basics of Computational Complexity Theory, Complexity of Local Optimization, Optimal Methods for Nonlinear Optimization.
7. Introduction to Linear Programming, the Simplex Method for Linear Programming, Duality and Sensitivity Analysis in Linear Programming.
8. Unconstrained Optimization.
9. Constrained Optimization.

Learning Outcomes

At the end of the course, the student will have to

- master the main numerical methods;
- understand the bases of linear programming, unconstrained optimization, constrained optimization;
- be able to analyze the behaviour of these numerical methods and in particular to be able to discuss their stability, their order of convergence and their conditions of application;
- be able to apply these methods to academic and simple practical instances;
- demonstrate the abilities to

  – apply knowledge of mathematics and computing to the design and analysis of optimization methods,

  – analyze a problem and identify the computing requirements appropriate for its solution,

– design and conduct experiments and numerical tests of optimization methods, and to analyze and interpret their results.

Projects

The goal of the project is to produce a coherent paper that explains a numerical implementation of some algorithms (Computer Assignments), and gives results. Neatness and notation is important.

Course Assessment

Note: Assessments subject to change. Below there is a tentative version of assessments. The final version will appear prior to start of the course.

Assessment Type	Number	Weight
Computer Assignments	5	50%
Midterm	1	25%
Final	1	25%

Attendance Policy

Students are expected to attend classes regularly, since the consistent attendance offers the most effective opportunity open to all students to gain command of the concepts and materials of the course.

References

1. M. S. Bazaraa, J. J. Jarvis, and H. D. Sherali. Linear Programming and Network Flows. John Wiley & Sons, 4th edition, 2010, ISBN 978-0-470-46272-0.
2. D. Bertsimas and J. N. Tsitsiklis. Introduction to Linear Optimization. Athena Scientific, Belmont, MA, 1997, ISBN 1886529191.
3. S. Boyd and L. Vandenberghe. Convex Optimization. Cambridge University Press, 2004, ISBN 0-521-83378-7.
4. G. Dahlquist and A. Bjorck. Numerical Methods. Dover Publications, 2003, ISBN 0-486-42807-9.
5. R. W. Hamming. Numerical Methods for Scientists and Engineers. Dover Publications, 1987, ISBN 0-486-65241-6.

Academic Honor Policy / Academic Honesty

The Siberian Federal University is built upon a strong foundation of integrity, respect and trust. All members of the university have a responsibility to be honest and the right to expect honesty from others. Any form of academic dishonesty is unacceptable to our community.

Note

The instructor reserves the right to make changes to this syllabus as necessary prior to start of the course.