Solving Polynomial Equations. Foundations and Algorithms

Course syllabus

Basic information

Program of study	Applied Computing in Engineering and Science (Master's Degree)
Semester	Third (Year 2 Fall Semester)
Course credits	3 ECTS
Language	English level B1 / Intermediate
Prerequisites	B. Sc. degree in Mathematics, Physics or Computer Science

Course Instructor

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Course description

The subject of the course is the solution of polynomial equations. It is at the heart of several areas of mathematics and its applications. The core of the subject is algebraic geometry, but it also calls upon aspects of combinatorics and function theory. The course is aimed to provide general introduction to modern mathematical aspects in computing with multivariate polynomials and in solving algebraic systems.

Required knowledge

The prerequisites include basic courses in commutative algebra, real and complex analysis. Programming skills (any high-level programming language) would help, but is not required.

Outline of content

1. Resultants and Discriminants in one variable: basic concepts and applications.
2. Combinatorial aspects of classical discriminant: Newton polyhedron, Newton diagram, amoeba of the algedraic set.
3. Multivariate resultants, A-discriminants.
4. Laurent polynomial systems: reduction algorithm, linearization, parametrization of the discriminant and solution.
5. Power series for general algebraic functions.
6. Mellin transforms in the theory of algebraic equations.

Learning Outcomes

Upon the successful completing the course student will be able to

- apply resultants in solving polynomial equations;
- use combinatorial tools in the study of discriminants ;
- explain the A-discriminant concept;
- solve algebraic equations using analytic methods;

- demonstrate the role of Mellin transforms in the theory of algebraic equations.

Teaching

The course consists of

- 36 hrs of lectures;
- 72 hrs of self-study time.

Home assignments (problem sets) will be assigned each week, to be turned in the following week. Review on theoretical material for self-study is assumed. At the end of the term the qualification (oral examination) will take place.

Special Features

The course is aimed to demonstrate the synthesis of algebraic and analytic approaches in solving of polynomial equations.

Course Assessment

Note: Assessments subject to change. Below there is a tentative version of assessments. The final version will appear prior to start of the course.

Assessment Type	Weight
Home assignments	30%
Review on theoretical material for self-study	20%
Oral examination	50%

Attendance Policy

It is important that students engage fully with all activities. Attendance is therefore regarded as essential.

Reading

1. B.L. van der Waerden, Algebra Volume I. Springer-Verlag New York, 1991, pp. XIV, 265.
2. David Cox, John Little, Donal O'Shea. Ideals, Varieties, and Algorithms. An Introduction to Computational Algebraic Geometry and Commutative Algebra. Springer-Verlag, New-York, 4th Edition, 2015.
3. Bernd Sturmfels. Solving Systems of Polynomial Equations, Amer.Math.Soc., CBMS Regional Conferences Series, No 97, Providence, Rhode Island, 2002.
4. Gunter M. Ziegler. Lectures on Polytopes, Graduate Texts in Mathematics 152, Springer-Verlag New York Berlin Heidelberg, Revised edition, 1998.

Academic Honor Policy / Academic Honesty

Siberian Federal University is built upon a strong foundation of integrity, respect and trust. All members of the university have a responsibility to be honest and the right to expect honesty from others. Any form of academic dishonesty is unacceptable to our community

Note

The instructor reserves the right to make changes to this syllabus as necessary prior to start of the course.