Analytic Index Theory

Course credits: 2 ECTS

About the course

The course is devoted to the index theorem for Toeplitz operators, which is a special and very fruitful case of the remarkable Atiyah–Singer index theorem.
The course consists of

- 38 hrs of lectures
- 34 hrs of self-study time

Outline of content

1. C^*-Algebras and Operator Theory.
2. Fredholm Operators and the Calkin Algebra.
3. Toeplitz Index Theorem.
4. Elliptic Complexes.

Educator

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Special Features

No

Prerequisites

The students will need to have completed a basic course in functional analysis, and have had some exposure to homology theory.

Course aims

The program aims to introduce students to the fundamentals of the index theory for elliptic operators on compact smooth manifolds without boundary. The index theory for elliptic operators combine many different mathematical fields such as functional analysis, partial differential equations, and topology.

Course objectives

1. To give students an introduction to the index theory of Toeplitz operators.
2. To make students familiar with the connections complex analysis has with the Fredholm operator theory.
3. To provide students with the opportunity to develop academic and research skills.

Learning Outcomes

On completion of this course, it is expected that students will be able to:

1. Recall the terminology of the Fredholm operator theory, basic definitions and statements.
2. Know basics of index theory for elliptic operators on compact smooth manifolds without boundary.
3. Understand the formulation of the Atiyah–Singer index theorem.
4. Prove rigorously mathematical statements and formulate precise mathematical arguments.

Syllabus

Week	Lectures	Self-study / Assignments	Hours
1-3	C^*-algebras and operator theory. Gelfand transform.	Compact operators. Hilbert–Schmidt spectral theorem.	6+6
4-7	Fredholm operators. Properties of analytic index. Atkinson's theorem.	Properties of Schatten ideals.	8+8
8-12	Elliptic pseudodifferential operators.	Distribution theory and Fourier transform.	10+10
13-15	Toeplitz extension. Toeplitz index theorem.	Basics of Toeplitz and Hankel infinite matrices.	6+6
16-18	Elliptic complexes.	Dolbeault and de Rham cohomology.	6+4
19	Summary.		2

Assessment

Homework assignments 20%
Examination 60%

Attendance policy

Students are expected to attend classes regularly, for consistent attendance offers the most effective opportunity open to all students to gain command of the concepts and materials of the course.

Textbooks:

1. Guillemin, V., Toeplitz operators in n dimensions, Integral Equations and Operator Theory **7** (1984), 145–205.
2. Higson, N., and Roe, J., Analytic K-homology, Oxford University Press, Oxford, 2000, 405 pp.
3. Shubin M. A., Pseudodifferential Operators and Spectral Theory, Springer-Verlag 2001. ISBN 3-540-41195-X.
4. Tarkhanov, N., The Cauchy Problem for Solutions of Elliptic Equations, Akademie Verlag, Berlin, 1995.