Integral Representations and Transforms

Course credits: 5 ECTS

About the course

The course focuses on methods of integral representations and transforms in complex analysis. It covers also their applications in theory of algebraic equations and quantum physics.
The course consists of

- 38 hrs of lectures
- 106 hrs of self-study time

Students will have an oral examination at the end of the course.

Outline of content

1. Integral Representations in Several Complex Variables.
2. Applications of Integral Representations to problems of analytic extension.
3. Foundations of Mellin Transform Analysis.
4. Mellin Transform in Theory of Algebraic Equations.
5. Mellin Transform in Quantum Field Theory.

Educator

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Special Features

Prerequisites

The prerequisites include basic courses in calculus, algebra, complex analysis in one variable and topics in several complex variables.

Course aims

- to give students an introduction to methods of integral representations in several complex variables
- to make students familiar with the Mellin transform analysis

Course objectives

- to acquaint students with different types of integral representations and model cases for their applications.
- to make students familiar with the role of integral representations in problems of analytic extension.
- to give students an introduction to the theory of Mellin transform (inversion theorems and fundamental correspondence)
- to develop skills in application of the Mellin transform techniques in the theory of algebraic functions and theoretical physics.

Learning Outcomes

By the end of the course, students will be able to

- distinguish special features of different types of integral representations;
- demonstrate applications of integral representations in solving of problems of analytic extension;
- explain the main concepts and relationships in theory of the Mellin transform;
- use Mellin transform technique in applied problems.

Syllabus

Week	Lectures	Self-study / Assignments	Hours
1-3	Cauchy Formula. Bochner-Martinelli Formula. Leray Integral Formula.	Weil Formula. Polynomial Approximation of Holomorphic Function. Runge Theorem. Andreotti-Norguet Formula.	
4-6	Tangent Cauchy-Riemann Equations. Bochner-Severi Theorem.	Theorems on Removal of Singularities.	
7-11	Mellin Transform Inversion Theorems in Single Variable. Multidimensional Mellin Transform. Fundamental Correspondence for Mellin Transform.	Inversion Theorems for the Multidimensional Mellin Transform. Asymptotics of Harmonic Sums.	
12-16	Mellin-Barnes Integral Representation for Solution to the General Algebraic Equation. Mellin Formulae for Superposition of General Algebraic Functions	The Convergence Set of the Mellin-Barnes Integral. Mellin-Barnes Integral Representations for Monomial Functions of Solutions to Polynomial System	
17-19	Perturbative approach. Non-perturbative asymptotic improvement. Mellin-Barnes Hyperasymptotic Procudure.	Borel Resummation	

Assessment

- Homework assignments 20 %
- Report on theoretical material for self-study 30%
- Oral examination 50%

Attendance policy

Students are expected to attend classes regularly, for consistent attendance offers the most effective opportunity open to all students to gain command of the concepts and materials of the course.

Textbooks

1. B.V. Shabat. *Introduction to Complex Analysis Part II. Functions of Several Variables.* AMS, Translations of Mathematical Monographs, V. 110, Providence, 1992.
2. L. Hörmander. *An Introduction to Complex Analysis in Several Variables.* North Holland, 1990.
3. G. M. Henkin. *The Method of Integral Representations in Complex Analysis.* Enc. Math. Sci. 7, Sev. Cpl. Var 1, Springer, Berlin etc., 57-116, 1990.
4. Philippe Flajolet, Xavier Gourdon, Philippe Dumas. *Mellin transforms and asymptotics: Harmonic sums*. Theoretical Computer Science 144 (1995), 3-58. http://www.sciencedirect.com/science/journal/03043975/144
5. Antipova Irina A. *Inversion of many-dimensional Mellin transforms and solutions of algebraic equations.* Sbornik: Mathematics (2007), 198(4):447-463.
6. R.B. Paris and D. Kaminski. *Asymptotics and Mellin-Barnes Integrals*. Cambridge Univ Pr, 2001.