Operator Equations in Hilbert Spaces

Course credits: 4 ECTS

About the course

The course is devoted to application of the standard theorems of functional analysis to the investigation of two typical problems of the Theory of Partial Differential Equations: the (well-posed) Dirichlet problem for strongly elliptic systems and the (ill-posed) Cauchy problem for elliptic systems.
The course consists of

- 38 hrs of lectures
- 70 hrs of self-study time

Outline of content

1. Operator Equations in Hilbert spaces. Normally solvable problems, Fredholm problems, ill-posed problems. Application of spectral theory to regularization of the problem.
2. The standard functional spaces and differential operators. Spaces of smooth functions, Hoelder spaces, Sobolev spaces.
3. The (well-posed) Dirichlet problem for strongly elliptic systems
4. The (ill-posed) Cauchy problem for elliptic systems

Educator

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Special Features

Prerequisites

The basic courses of Algebra, Mathematical Analysis (Calculus), Functional Analysis of bachelor and master levels (metric spaces, normed spaces, Euclidean spaces), Closed Graph Theorem, Fredholm Theorems, Spectral Theorems for compact and bounded self-adjoint operators in Hilbert spaces.

Course aims

1. To give students an introduction to applications of Functional Analysis in Modern PDE's
2. To make students familiar with the connections between Analysis, Algebra, Partial Differential Equations, Complex Analysis and Geometry.
3. To develop research competences in the frame of the Operator approach to solving boundary value problems for elliptic equations and systems

Course objectives

1. To give students the basic notions and theorems related to standard Hoelder and Sobolev spaces.
2. To make students familiar with the basic theorems related to Elliptic equations and systems in standard (Hoelder and Sobolev) spaces .
3. To form an experience of studying Fredholm boundary value problems,
4. To form an experience of studying ill-posed problems

Learning Outcomes

The student will

1. know the basic notions and theorems of the Operator Theory in Banach and Hilbert spaces,
2. be able to see the difference between the ill-posed and well-posed problems for linear bounded operators
3. be able to apply the obtained knowledge to investigation of the standard Fredhoolm boundary values problems for elliptic systems in Sobolev spaces
4. be able to apply the obtained knowledge to the regularizaion of the standard ill-posed problems for elliptic systems

Syllabus

Week	Lectures	Self-study / Assignments	Hours
1-2	Operator Equations in Hilbert spaces. Normally solvable problems, Fredholm problems, ill-posed problems. Application of spectral theory to regularization of the problem	Hadamard's Example for the Cauchy problem for the Laplace operator, The Dirichlet problem for the Laplace equation	4+8
3-6	The standard functional spaces and differential operators. Hoelder spaces. Sobolev spaces, Embedding Theorems, trace theorems.	Spaces of smooth functions.	8+10
7-14	The (well-posed) Dirichlet problem for strongly elliptic systems. Differential operators with injective symbol, elliptic operators. The first Green formula for differential operators. Garding inequality for strongly elliptic systems. A priori estimates and Regularity Theorems for elliptic systems. The weak formulation of the Dirichlet problem for strongly elliptic systems. Hodge Theorem for the Dirichlet problem. Fundamental solutions to elliptic	Properties of the Fourier transfrom. The classical formulation of the Dirichlet problem for strongly elliptic systems, the Cauchy-Green formula for the Cauchy-Riemann operator, the Martinelli-Bochner formula for the multi-dimensional Cauchy-Riemann operator, the Green formula for the Laplace operator	16+32

	systems, the second Green Formula, examples (the Somigliana formula for the Lame operator).		
15-19	The (ill-posed) Cauchy problem for elliptic systems. The Cauchy problem for holomorphic functions. The Cauchy type integral: the boundary behavior and the Jump Theorem, the Cauchy problem and the problem of the analytic continuation. Bases with double orthogonality and the Cauchy problem, the abstract Carleman Formula, examples.	The analytic continuation along a line, the Sokhotskii Theorem for the Cauchy type integral	10+20

Assessment

Attendance 5%
Homework assignments 10%
Examination 85%

Attendance policy

Students are expected to attend classes regularly, for consistent attendance offers the most effective opportunity open to all students to gain command of the concepts and materials of the course.

Textbooks:

1) A.N. Kolmogorov, S.V., Fomin Elements of Functions Theorem and Functional analysis, Dover books on Mathematics, 1999.

2) V.A. Trenogin, Functional Analisis, Nauka, Moscow,1980.

3) Gilbarg, D., Trudinger, N. Elliptic Partial Differential
Equations of second order, Berlin, Springer-Verlag, 1983.

4) V.P. Mikhailov, Partial Differential equations. Nauka, Moscow,1986.

5) N. Tarkhanov, N. The Cauchy Problem for Solutions of Elliptic Equations,
Berlin: Akademie-Verlag, 1995.