## **Basic Information**

#### This is a course, which contributes to MSc award in Mathematics

| Duration                 | 216 hours (6 ECTS)  |  |
|--------------------------|---|--|
| Starting date            | September 1st   |  |
| Study credits            | 6 ECTS credits  |  |
| Language of instruction  |   |  |
| Academic<br>requirements | <ul> <li>BSc degree in Mathematics, Physics, Computer Science, Engineering or equivalent (a copy of your diplomas from previous university studies and transcripts of completes courses and grades)</li> <li>Skype interview</li> </ul> |  |

#### **Course Description**

A typical method to solve various numeric problems in science and technology is to linearize them, that is to reduce them to numeric problems in linear algebra. The course "Numerical Linear Algebra" focuses on the latter problems. This course provides the base for future courses devoted to numerical analysis.

We discuss linearization of problems, statement of linear numeric problems, modern approaches and algorithms to solve them and theoretical methods to analyze the problems and solutions.

### **Special Features of the Course**

The aim of the course is to immediately enable the students to employ numeric algorithms in their investigations. For that sake we try out the algorithms in programming environments, such as Matlab/Octave, Maxima, depending on which is more appropriate for a particular problem.

A lot of technical details of robust implementation of the algorithms are discussed.

Along with classical algorithms, the course also includes different enhancements to target ill-conditioned or ill-posed problems.

## **Course Aim**

The course aims to give students the skills and theoretical basis to learn and investigate the existing approaches and algorithms in numerical linear algebra, as well as to develop new numeric algorithms to solve the problems they face in their areas of science and technology.

# **Course Objectives**

- To acquaint the students with the theory behind the numerical linear algebra and classical as well as modern algorithms in numerical linear algebra;
- to acquaint the students with modern software environments and libraries to solve numerical linear algebra problems;
- to acquaint the students with modern trends in numerical linear algebra;
- to develop practical skills in programming numerical linear algebra algorithms.

# Learning Outcomes of the Course

By the end of the course, students will be able to:

- chose appropriate algorithms for solving a particular linear numerical problem;
- solve numerical linear algebra problems by means of existing software;
- estimate the expected accuracy of the solution to a problem and estimate the actual accuracy of the solution obtained;
- identify possible problems in algorithms in numerical linear algebra, propose improvements;
- develop improvements to algorithms to account the peculiarities of a particular problem.

| Learning Activities                | Hours |
|------------------------------------|-------|
| Lectures                           | 18    |
| Practice sessions / Seminars,      | 36    |
| Self-study Assignments             | 126   |
| Final Exam (including preparation) | 36    |
| Total study hours                  | 216   |

## **Course (module) Structure**

## **Course Outline**

| Week | Lectures       | Practice session / Assignments   | Hours <sup>1</sup> |
|------|----------------|--|--------------------|
| 1-2  | Preliminaries  | <ul> <li>Machine number representation;</li> <li>Basic linear algebra background;</li> <li>Home assignment No 1;</li> <li>Home assignment No 2.</li> </ul>   | 20                 |
| 3-7  | Direct methods | <ul> <li>LU decomposition;</li> <li>Backward substitution;</li> <li>Condition number of a triangular matrix;</li> <li>Householder and Givens transformations;</li> <li>Gramm-Schmidt orthogonalization;</li> <li>QR decomposition;</li> <li>Cholesky decomposition;</li> </ul> | 50                 |

<sup>&</sup>lt;sup>1</sup> Hours designed for Classroom sessions, Web-sessions, Home Assignments etc.

|       |                        | <ul> <li>Bidiagonal decomposition;</li> <li>Stability analysis;</li> <li>Home assignment No 3;</li> <li>Home assignment No 4.</li> </ul>   |    |
|-------|------------------------|--|----|
| 8-10  | Eigenvalues problems   | <ul> <li>Power iteration;</li> <li>Simultaneous iteration;</li> <li>QR iteration;</li> <li>Implicit QR iterations;</li> <li>QR iteration with shifts;</li> <li>Home assignment No 5.</li> </ul>  | 30 |
| 11-14 | Iterative methods      | <ul> <li>Classical iterative methods overview;</li> <li>Arnoldi and Lancoz iteration;</li> <li>Conjugate gradient iteration;</li> <li>Preconditioning;</li> <li>Home assignment No 6.</li> </ul>   | 40 |
| 15-18 | Least squares problems | <ul> <li>Pseudoinverse matrix;</li> <li>Pseudorank of a matrix;</li> <li>Various forms of least squares problems<br/>(full-rank, rank deficient, constrained<br/>problems);</li> <li>Approaches to ill-conditioned and ill-posed<br/>problems;</li> <li>Home assignment No 7.</li> </ul> | 40 |
| 19-20 |                        | Final exam   | 36 |

#### Assessment

Grade policy for both practical home assignments and the final exam is:

- A (excellent work) 91–100 points
- B (above average work) 81–90 points
- C (average work) 71–80 points
- D (below average work) 50–70 points
- F (failed work) < 50 points

The exam is taken orally. Each exam ticket consists of 2 theoretical questions from the list of exam questions.

The final grade includes grade for homework and exam in a ratio of 50:50.

## **Attendance Policy**

Students are expected to attend classes regularly. However, occasional skipping classes is permissible if a student does all necessary in-class work at home.

## Lecturer(s) and Tutors, Contact Information

#### Yuriy USHAKOV

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# **Core Reading**

All the books are available on-line in Elsevier at SFU library.

William Ford, Numerical Linear Algebra with Applications, 1st Edition, Using MATLAB, Hardcover ISBN: 9780123944351, eBook ISBN: 9780123947840, Imprint: Academic Press, Published Date: 2nd September 2014, Page Count: 628